TIDAL THEORY

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INTRODUCTION

The word "tides" is a generic term used to define the alternating rise and fall in sea level with respect to the land, produced, in part, by the gravitational attraction of the moon and sun. There are additional nonastronomical factors influencing local heights and times of tides such as configuration of the coastline, depth of the water, ocean-floor topography, and other hydrographic and meteorological influences.

Knowledge of these heights and times is important in practical applications such as navigation, harbor construction, tidal datums for hydrography and for mean high water line survey procedures, the determination of base lines for fixing offshore territorial limits and for tide predictions.

ORIGIN OF ASTRONOMICAL TIDE RAISING FORCES

At the surface of the earth, the earth's force of gravitational attraction acts in a direction toward its center, and thus holds the ocean waters confined to this surface. However, the gravitational forces of the moon and sun also act externally upon the earth's ocean waters. These external forces are exerted as tide producing forces, called tractive forces. To all outward appearances, the moon revolves around the earth, but in actuality, the moon and earth revolve together around their common center of mass called the barycenter. The two astronomical bodies are held together by gravitational attraction, but are simultaneously kept apart by an equal and opposite centrifugal force. At the earth's surface an imbalance between these two forces results in the fact that there exists, on the side of the earth turned toward the moon, a net tide producing force which acts in the direction of the moon. On the side of the earth directly opposite the moon, the net tide producing force is in the direction of a greater centrifugal force, away from the moon.

THE TIDAL FORCE ENVELOPE

If the earth were continent free and if it were covered with a uniform depth of water, the ocean waters would respond completely to the directions and magnitudes of the tractive forces of the moon and sun. A mathematical figure would be formed having the shape of a prolate spheroid. The longest (major) axis of the spheroid would extend toward and directly away from the moon, and the shortest (minor) axis would be centered along, and at right angles to, the major axis. The directions of the major axis and rotated minor axis of the spheroid would represent two tidal humps and two tidal depressions in this force envelope, respectively. From a purely theoretical point of view, the daily rotation of the solid earth with respect to these two tidal humps and two depressions would be the tides.

The earth rotates once in 24 hours. Thus one would ideally expect to find a high tide followed by a low tide at the same place 6 hours later, then a second high tide after 12 hours, a second low tide 18 hours later, and finally a return to high water at the expiration of 24 hours. Such, in actuality, is far from the situation that exists.

First, a tidal force envelope produced by the sun accompanies this tidal force envelope produced by the moon. The tidal force exerted by the sun is a composite of the sun's gravitational attraction and a centrifugal force component created by the revolution of the earth's center-of-mass around the center-of-mass of the earth-sun system, in an exactly analogous manner to the earth-moon relationship. The position of this force envelope shifts with the relative orbital position of the earth in respect to the sun. Because of the great difference between the average distances of the moon (238,855 miles) and the sun (92,900,000 miles) from the earth, the tide-raising force of the moon is approximately 2 1/2 times that of the sun.

Second, there exists a wide range of astronomical variables in the production of the tides. They are: the changing distances of the moon from the earth and the earth from the sun, the angle which the moon in its orbit makes with the earth's equator, the superimposition of the sun's tidal envelope of forces upon that caused by the moon, the variable phase relationships of the moon, etc.

VARIATIONS IN RANGE AND INEQUALITIES OF TIDES

The difference in height between consecutive high and low tides occurring at a given place is known as the range. The range of the tides at any one location is subject to many variable factors. The following are those of astronomical origin.

1. Lunar Phase Effects: Spring and Neap Tides. Since the gravitational forces of both the moon and the sun act upon the waters of the earth, the moon's changing position with respect to the earth and sun, during its monthly cycle of phases, effect the tidal range. The phase cycle is called a lunar month or synodical month and has a period of about 29.53 days. The gravitational attraction of the moon and sun act along a common line at changing angles relative to each other.

When the moon is at new phase and full phase, both called syzygy, the gravitational attraction of the moon and sun act to reinforce each other and the resultant or combined tidal force is also increased. Observed high tides are higher and low tides are lower than average making the tidal range greater than average. These tides are known as spring tides from a Middle English word, *springan*, which implies a "springing up" of the water and bears no relationship to the season of the year.

At first and third-quarter phases of the moon, called quadrature, the gravitational attraction of the moon and sun are exerted at right angles to each other. Each force tends, in part, to counteract the other. The resultant or combined tidal force is thereby reduced. High tides are lower and low tides are higher than average making the tidal range lesser than average. These tides are called neap tides, from an Old English word *nepflod* and Middle Dutch word *nipen* "to bite". The tides are "nipped off" so to speak.

2. Parallax Effects: Moon and Sun. Since the moon follows an elliptical path around the earth, the distance between the earth and moon will vary throughout the month by about 31,000 miles. The moon's gravitational attraction for the earth's waters will change with the change in the distance between earth and moon. Once each month, when the moon is closest to the earth, at perigee, the tide-generating forces will be higher than usual, thus producing above-average ranges in the tides. Approximately two weeks later, when the moon is farthest from the earth, at apogee, the lunar tide-raising force will be smaller and the tidal ranges will be less than average.

Similarly, in the sun-earth system, when the earth is closest to the sun, at perihelion, about January 2nd of each year, the tidal ranges will be enhanced, and when the earth is farthest from the sun, at aphelion, around July 2nd, the tidal ranges will be reduced. These dates are near the sun's maximum declination at winter solstice, on or near December 22nd, and at summer solstice on or near June 21st of each year.

When perigee, perihelion, and either new or full moon occur at approximately the same time, considerably increased tidal ranges result. When apogee, aphelion, and first or third-quarter moon coincide at approximately the same time, considerably reduced tidal ranges will occur.

3. Lunar Declination Effects: The diurnal Inequality. The plane of the moon's orbit is inclined about 5° to the plane of the earth's orbit and thus the moon in its monthly revolution around the earth remains close to the ecliptic. The ecliptic is the intersection plane of the earth's solar orbit with the celestial sphere and is inclined 23 1/2° to the earth's equator, north and south of which the sun moves once each half year to produce the seasons. This is called a tropical year and has a period of 365.2422 days. This agrees very closely to our Gregorian calendar year of 365.2425 days.

The moon, in making a revolution around the earth once each month, passes from a position of maximum angular distance north (23 1/2° +/- 5°) of the equator to a position of maximum angular distance south (23 1/2° +/- 5°) of the equator during each half month. This is called a tropical month and has a period of about 27.32 days. This angular distance is termed lunar declination and twice each month the moon crosses the equator. The cycle of variation from 18 1/2° to 28 1/2° and back again is called the node cycle and has a period of 18.61 years to complete a circuit of 360 degrees of longitude. This results in a small inequality in the rise and fall of the tide.

METONIC CYCLE

This cycle has a period of almost 19 years or 235 lunations. Devised by Meton, an Athenian astronomer who lived in the fifth century B.C., for the purpose of obtaining a period in which new and full Moon would recur on the same day of the year. Taking the Julian (solar) year of 365.2422 days and the synodic (lunar) month as 29.530,588 days, we have a 19-year period of 6,939.60 solar days as compared with the 235 lunations of 6,939.69 lunar days, a difference of only 0.09-day. This period also contains one node cycle of 18.61 Julian years or 6,797.16 days, plus about 2% of the next node cycle (an insignificant factor in tidal amplitude). This 19-year period also contains 19 annual inequalities in the tide due to meteorological factors such as wind and pressure. These are factors influencing local heights and times.